1.0 Perfect Purple Paint (Ratios and Proportions)

“Ratios and proportions are foundational to student understanding across multiple topics in mathematics and science. In mathematics, they are central to developing concepts and skills related to slope, constant rate of change, and similar figures, which are all fundamental to algebraic concepts and skills. Ratios and proportions are used in relationships found in triangles, including trigonometric ones, such as sine, cosine, and tangent, found in later algebraic instruction. In science, they are used when quantities involve density, acceleration, and other comparable derived measures. Even in real-life situations, ratios and proportions are useful when determining amounts to be used in recipes or finding the mileage per gallon of gas. In general, ratios and proportions describe relationships between and among quantities.”

- *Helping Students With Mathematics Difficulties Understand Ratios and Proportions. Barbara Dougherty, Diane Pedrotty Bryant, Brian R. Bryant, and Mikyung Shin. TEACHING Exceptional Children, Vol. 49, No. 2, pp. 96–105. Copyright 2017*

In this section, students develop an understanding of the concept of ratios and connect the numbers to the graph of the data. They can begin to understand the special proportional relationship that occurs when they graph the equivalent ratios and how to identify it as a straight line that passes through (0,0) and what makes it different from fractions or more random data. They are developing the understanding to “develop” a formula for linear equations.

Instructional Goals:

- **6th Grade**: Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed.
- **7th Grade**: Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of

proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations.

- **8th Grade**: Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. . *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- **8th Grade**: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Section 1.0 - Perfect Purple Paint (Ratios and Proportions)

1.1 Introduction
1.2 Create a Model
1.3 Refine Your Model (Whiteboard Discussion)
    Suggested Assignment: Intro to Ratios
    Suggested Assignment: Help Nico with Ratios
1.4 Practice Help Giving & Review Talk Moves
1.5 Create Your Own Color
1.6 Discuss your Model
1.7 Integrate Feedback on Model
    Suggested Assignment: Proportional Relationships
    Suggested Assignment: Solving Proportions

Suggested Lesson Breakdown

Pre-work: Introducing Help-giving, modeling, and class norms	**PPP: Unifix Whiteboard Activity**	**PPP: Paint Splash Whiteboard Activity**
Pretest (45 min)	**1.2 Create a PPP Model**	**1.4 Practice Help Giving & Review Talk Moves** Help Giving Assignment 1 (independent work of homework) and review
1.1 Introduction: Welcome, class norms, homework/grading explanation Introduction to Modeling	**1.2 Review other PPP Models** Gallery Walk	**1.5 Create Your Own Color**: Phet Simulation - Paint Splash
Introducing Help Giving (Talk moves expectations and assignment) (Independent)	**1.3 Refine Your PPP Model:** Whiteboard Discussion	**1.6 Discuss Your Model**: Small group chat
	Suggested Assignment: Intro to Ratios	**1.7 Integrate Feedback on Paint Splash Model:** Whiteboard Meeting
	Suggested Assignment: Help Nico with Ratios Assignment	**Suggested Assignment: Proportional Relationships Assignment**
	1.0 Full Packet of Student Assignments	**Suggested Assignment: Solving Proportions Assignment**

Pre-work: Introducing Help-giving, modeling, and class norms

Lesson	Text	Teacher Directions	Materials
PPP 1.1	**1.1 Introduction - Start here!** In the next three weeks, you will be interacting with your fellow classmates and others to learn and help teach some math concepts. We want to help you become better collaborators so: - You can support each other in your learning. - You can become more active learners. - You can construct your own STEM knowledge.		
Pretest	**Pretest**	You can use either of the tests linked here as a pretest and use the other test as a post-test.	Test Option 1 Test Option 2
PPP 1.1	**1.1 Class Norms** Let's have a discussion about your classes. How do you like class to work? Let's discuss. Now that we know your class norms and what you want from me, let's take a look at what we need from you in a modeling classroom.	Review your own class norms with the class (if you have them) and use the left side of the provided Norm slide to document expectations. If you don't have class norms, take this time to document expectations or feel free to use the provided ones as an example. Set behavior expectations. Ask the students what they want from you,	Image of group norms Slides for Class/Study Norms

		and write it on the left side of the slide before they click on the slide Let them know what you want from them. On the right side of the norms slide, we have modeling norms that you should feel free to use alongside the sample image of group norms.	
PPP 1.1	**1.1 Introduction to Modeling**	Go through the linked guide to modeling with your students Review the teacher introduction if needed	1.0_Intro to Modeling for Students Modeling intro for teachers
PPP 1.1	**1.1 Introducing Help Giving** It's really important to learn to help others. Not just for others learning, but yours too. Now is your chance to get familiar with some help-giving techniques. Complete the provided Talk Moves assignment. **After Assignment** Do you have any questions about the Help-Giving and Talk Moves Assignment? Do you have any questions about anything so far?	In the help giving scenarios assignment, students pick best move and fill in an answer **Conversation starter:** "What did you think the best answer was for the first scenario?" **Expectations:** There is no one "best" talk move for a particular situation. **Conclude there is no one answer.** You can be helpful without completely	1.0_Talk Move Assignment Talk moves teacher sample Talk move student sample

	Now's your chance to discuss! Write your questions for the class to read (see digital/physical instructions in introduction for clarification). Look at the questions. Are there any questions posted about the Help-Giving and Talk moves Assignment? Can you give any help to your classmates?	knowing the content. You can review sample talk moves and their goals in the teacher copy and, if you'd like, distribute a student copy to your students You can save part of this assignment to review when you circle back to talk moves in PPP: Paint Splash Whiteboard Activity	

PPP: Unifix Whiteboard Activity

Lesson	Text	Teacher Directions	Materials
PPP 1.2	**1.2 Create a PPP Model** I've found the perfect color to paint my house. It's made by mixing two cups of blue paint with three cups of red paint. That makes one batch of Perfect Purple Paint. I know it will take at least 20 cups to paint my bathroom. I don't want to make a bunch of small batches. How much of each color will I need to paint the entire bathroom? Let's make a model using Unifix Cube blocks to represent the information in this problem.	Group work, break up students responsibilities as follows (specifics related to digital/physical learning provided in the introduction): • Recorder • Presenter • Leader **In-person:** If you have physical unifix cubes, you can conduct the activity with those. Otherwise, you can have students conduct the experiment on classroom devices (either one per student or per group).	1.0_Unifix cubes online Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard option 3 -

	Have one color block represent red paint and another color block represent blue paint. Show how you could make 20 cups of purple paint using the Unifix tool. **Explain your PPP Model** Now your group will use a whiteboard to explain your thinking. Everyone needs to agree with and understand everything that goes on the group board. As you make your whiteboard, think about these questions: ● How do you know the color will be the same in the small batch as it is in the larger batch? ● Can you find multiple ways to model this relationship? What if we only needed enough paint for one purple accent wall? ● Can you write a rule (using words or numbers) to create this color of paint that anyone could follow for any number of cups? ● Explain your thinking in words and draw arrows or labels to help others understand your thinking	**Digital:** Have students conduct the experiment on their personal devices using the unifix link. **Review Ratio Misconceptions:** ● Ratios amounts are often confused with fractions involving the same digits. For instance 2 : 3 is confused with ⅔ or 1 : 2 = ½. ● When solving problems involving proportion students tend to struggle with forming a ratio. For instance, 3 apples cost 45 cents would form the ratio apples : cost. ● When writing ratios into the form 1 : n students incorrectly assume that n has to be an integer or greater than 1. ● Ways to represent ratios: Drawing, using colon (:), the word “to”, ● Does not mean ⅔ two out of three. Not a fraction!!! This would mean 2 out of the three cups, not 2 cups and 3 cups (total 5.) Teacher visits each group. For	Miro’s Web White Board

		struggling students ask: • Can you reduce ratios in the same way you reduce fractions? What's the difference? • Can you multiply the ratio and still keep the same color? For students that already understand ask: • Can you write a number sentence or equation that would work for any amount of paint? Your job is to prove that the color is consistent. How can you do that?	
PPP 1.2	**1.2 Review Other PPP Student Models** Gallery Walk: Look at other people's boards and leave constructive comments or ask helpful questions about other models. When the Gallery walk is over, each group needs to read and discuss their comments and be ready to answer your classmates' questions. Are there any changes you would like to make to your board before you defend your work?	You can **review talk moves** before you start this exercise and have a discussion similar to the day before. Model commenting for students: have students walk around and leave sticky notes on the whiteboards with comments and questions.	Talk move student sample If you're working digitally, use one of the provided tools to review other student's models and leave comments.

PPP 1.3	**1.3 Refine Your PPP Model** Let's discuss your whiteboards as a class in a "Board Meeting." Keep in mind the suggestions from the badges to ask good questions and make productive comments. 1. Would it help us to be able to visualize these numbers? What are ways we can see what's happening with data? 2. What is our data telling us? 3. How can you prove to me that it will make the same color? 4. Can we write a rule for this in words? We want to make sure we all agree on the terms we are using. Are there any words we need to define as a class? We can discuss terms using the **Vocabulary** section of the Model-So-Far page.	**Reminder:** DON'T Pre-define vocabulary - use another whiteboard in the meeting or the model-so-far slide to define vocabulary together. **Teacher Led Discussion:** ● (If doing this digitally, teacher shares screen) ● Discuss Ratio vs fraction ● Use talk moves from earlier in class ● Teacher graphs different student responses. Observe the linear relationship when the equivalent ratios are graphed. **Goal**: Identify/solve/define ratios **Whiteboard meeting questions:** What similarities do we see between all the groups? (ex. one goes up the other goes up, same ratio/reduces to the same number.) ● What is our data telling us? ● How can you prove to me that it will make the same color? When you graph the students' ratios, it will become clear when one of the data points is not on the line. A class graph	Digital graphing tool for linear equations Model-So-Far page for vocabulary

		of the data will be part of the proof that the color will be the same. (See below) Use one exemplar whiteboard to show table, graph, sketch, description ● Discuss unit rate ● Write a rule in words We seemed to have noticed there is a special relationship between the red and blue paint. They all reduce to ⅔. Anyone seen a relationship like this before? This relationship is called a proportion. Based on our observations, what do we think are the characteristics of a proportion? How can we make more sense of our numbers (elicit response: make a table, make a graph.) Draw out the importance of using a diagram, labels, and explanation to get their point across. <u>Use a whiteboard to create a graph that uses the ratios from every group. This will make the linear relationship created by the ratio with the data the students calculated.</u>	

		Ask students: Looking at this graph of our data, if I had 9 cups of blue paint how many red cups would I need to get my perfect purple color? (Use some #'s that aren't already plotted) (Have students sketch a conceptual graph on their whiteboard to refer back to later) You can have students take a photo of the whiteboard and add it to a google sheet to have easy access to all boards for reference later. You can have students erase all whiteboards at this time.	
Suggested Assignment	**Suggested Assignment: Intro to Ratios** If you need help with ratios, watch the Khan Academy Finding Ratios: An Introduction	If your students need help with ratios, you can tack on the Khan Academy video as homework or classwork. **Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	1.0_Khan Academy - Finding Ratios: An Introduction 1.0_Ratios Assignment

	Suggested Assignment: Help Nico with Ratios Assignment When you are teaching someone else, it is often helpful to know the process and the answers before you begin. Use this assignment to be ready to defend your answers. You can also review **Talk Moves** to help you make constructive comments.	This assignment heavily scaffolds students through the process of solving the ratios while allowing them to practice help giving with talk moves.	1.0_Help with Ratios Assignment

PPP: Paint Splash Whiteboard Activity

Lesson	Text	Teacher Directions	Materials
PPP 1.4	**1.4 Practice Giving Help and Review Talk Moves: Assignment** Do you have any Questions about the Help Giving and Talk Moves Assignment? Do you have any questions about anything so far? Now's your chance to discuss! You can also review **Talk Moves** to help you make constructive comments.	You can save a few of the scenarios from the help giving assignment and circle back here to reinforce and remind students about the help giving and dialogue techniques. Review Help Giving and Talk Moves have students defend their choices, but be sure to reinforce the understanding that there is no "wrong" choice if it helps their	1.0_Talk Move Assignment Talk moves teacher sample Talk move student sample

		understanding or the understanding of the class. Conversation starter for the discussion: "What did you think the best answer was for the first scenario?" You can use the Bellwork slides to assist in your discussion. **Conclude there is no one answer.** You can be helpful without completely knowing the content.	
PPP 1.5	**1.5 Create Your Own Color**: **Phet Simulation - Paint Splash** I love my Perfect Purple Paint, but you probably want to come up with your own awesome color for your house. Check out Paint Splash here: https://phet.colorado.edu/en/simulation/proportion-playground Working as a group in Paint Splash in the single paint splash mode, create a paint splash of your choice. Your group needs to agree on the color. Each group member should then use the double paint splash setting to recreate your group's paint splash on the left. And then	Students will agree on a group color and create a table and graph. For group work, break up students responsibilities as follows (specifics related to digital physical provided in the introduction): • Recorder • Presenter • Leader **In-person:** Have students conduct the experiment on classroom devices (either one per student or per group). **Digital:** Have students conduct the experiment on their personal devices using the unifix link.	1.0_Proportion Playground - Paint Splash Digital graphing tool for linear equations Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard

	on the right create either a larger or smaller matching batch (each group member should create a different sized matching paint splash on the right). Enter all the group members' Ratios into the graphing tool. Remember, your whiteboard needs: • A Title • A picture of one group member's double paint splash (2 different sized batches) • A picture of the table that lists your group members' batches • A picture of your graph • A written explanation about how you know how to find any size batch for your color. • A rule in words and numbers.		option 3 - Miro's Web White Board
PPP 1.6	**1.6 Discuss Your Paint Splash Model: Gallery Walk** Look at other student's whiteboards and leave a comment on each picture. Ask a question for clarification, give a compliment, or explain what you are thinking. Be specific.	**1. Comments and Questions** Ask a Question Ask for clarification: "What do you mean by that?" "Can you give an example?"	Talk moves teacher sample Talk move student sample

	Review these **Talk Moves** to help you make constructive comments Do you understand / agree with their solutions? Use the talk moves to be clear and polite. Once you have commented individually, move to the group discussion: With your classmates, discuss the similarities and differences between the way groups represented the relationship between the red and blue paint? Does the way the group represented it change the color of the paint? How did people represent their perfect purple paint? Do different people's representations mean the same thing, and how? How are the representations (drawings, numbers, etc.) related?	Teacher makes comments to challenge the quality of student comments. **2. Analysis** If the way you write the ratio changes the color, what kind of rule can we define then it's not the same ratio. **Prompt -** Discuss the similarities and differences between the way groups represented the relationship between the color of the paint? Does the way the group represented it change the color of the paint? When students finish their discussion, have students look at the comments others made on their board. Ask: Do you want to make any changes before a Whiteboard meeting? Prompt - What are the similarities and differences between the graphs? What features do they share? Why do you think they share these features	

PPP 1.7	**1.7 Integrate Feedback on Model: Paint Splash Whiteboard Meeting** Review the comments your group received in the Whiteboard. Is there anything you would change on your Whiteboard Tool based on your classmates' feedback? What can you change to make your board more clear? If you do not yet have a rule on your board, do your best to add one. Think about how you would find the number of cups of each color needed for any size room. Let's discuss your whiteboards as a class. Keep in mind the suggestions from the talk moves to ask good questions and make productive comments. Make changes your group can all agree on. **Vocabulary** We want to make sure we all agree on the terms we are using. Are there any words we need to define as a class? We can discuss terms using the **Vocabulary** section of the Model-So-Far page.	While students are reviewing comments, try to encourage them to pull out the difference between a ratio and a fraction in the discussion. **Whiteboard meeting questions:** ● What similarities do we see between all the groups? (ex. one goes up the other goes up, same ratio/reduces to the same number.) ● What is our data telling us? ● How can you prove to me that it will make the same color? Project one exemplar whiteboard to show table, graph, sketch, description ● Discuss unit rate ● Write a rule in words There is a special relationship between Any color chosen. Has anyone seen a relationship like this before? All the quantities are proportional. Based on our observations, what do we think are the characteristics of a proportion? (you go up and over by the same amount on the graph, multiply by the unit rate, line passes through 0, line of a graph showing proportional	Talk moves teacher sample Model-so-far vocabulary Page

		relationship is a straight line) How can we make more sense of our numbers (elicit response: make a table, make a graph.) **Draw out the importance of using a diagram, labels, and explanation to get their point across.** DON'T Pre-define vocabulary (straight line/linear, goes through 0, same slope, same ratio, same unit rate) So what is the unit rate that we're seeing? Vocabulary to pull out: ratio, unit rate, proportion, (keep adding to definition in vocabulary brainstorm page as students make more connections. They don't need to see every piece at once.) A ratio is a comparison of two numbers A proportion is an equation that says two ratios are equivalent. Q: What does this table mean? What does the graph mean? A: The table is a list of ratios. The graph is a way of showing the	

		equation the says they are related. Explain why all the graphs deal with the first quadrant but omit the other three quadrants. Q: Why have we chosen to organize our data in this way? How does this help us understand our data? A: “Making Predictions” Q: How do you know if you’re looking at a graph of a proportional relationship? A: Passes through origin (0,0), straight line, when one goes up the other goes up. You don’t need to get students to a perfect understanding of proportion by the end of the WB meeting!!!	
Suggested Assignment	**Suggested Assignment: Proportional Relationships Assignment** We have just graphed two Proportional relationships. Watch the video on Proportional Relationship Assignment. Complete the assignment after watching this video on Khan academy to help you think about proportions.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else’s question.	1.0_Proportional Relationship Assignment 1.0_Khan Academy - Introduction to Proportions

Suggested Assignment	**Suggested Assignment: Solving Proportions Assignment** Watch the video about Solving Proportional Relationships. Complete the assignment after watching this video on Khan academy to help you solve proportions with a missing variable.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	1.0_Solving Proportions Assignment 1.0_Khan Academy - Solving Proportions