Middle School Math Modeling Curriculum For In-person, Digital, or Hybrid learning

Topics covered: Ratios & Proportions, Slope & Speed, and Linear Equations

Curriculum Overview

In order to understand linear equations, students need to have an understanding of the meaning of all the parts of the equation as well as how changes to the equation affect the graph of the equation and the meaning that the graph conveys. By starting with Ratios and Proportions, students are able to differentiate between these two very important mathematical concepts and determine which they are dealing with when they get to linear equations. Thay can begin to internalize what the ratios are representing in the real world and how changing the ratio changes the graph of the line of the graph and what it represents.

In the next section, students connect the proportional relationship of constant speed with the slope of the line. They can use the slope of a line to create the equation for the graph of their real world data and then use slopes to graph other lines and begin to understand the story of the graph even without having to collect the data themselves. At this point, students will have an understanding of “m,” the first part of the linear equation.

In Part 3, students will again use motion to give them context for the Y-intercept part of the linear equation. By knowing that the y-intercept can mean the startinging point, students can combine their understanding of the ratio as the slope of the line, model the proportional relationship on the graph and choose an appropriate starting point for data they are given, data they collect, or a graph they are provided with.

They can tell the story of what is happening in the graph by understanding the numbers in the equation and create an equation that mathematically explains the motion described by the graph.

Options for In-Person and Digital Learning

Throughout this curriculum, we're offering options for conducting classroom activities or discussions both in-person and online, which we hope will be especially useful given the uncertainty of teaching with the ongoing COVID-19 pandemic. We want to provide options for each step of the activities that typically appear in this curriculum.

The steps in each lesson usually take the following form:

1. Conducting a lab
2. Creating a whiteboard in small groups
3. Commenting and discussing the class' whiteboards
4. Having a class-wide whiteboard meeting

Here, we break down what each step looks like in-person or digitally.

	In Person	**Digital**
Conducting a lab	Some labs are best conducted digitally, others physically, and others could involve either. We include options in the curriculum for both digital and physical experiments. In each section, you'll find recommendations for conducting the lab depending on which modality you're working in.	
Creating a whiteboard in small groups	Create physical whiteboards and ask students to draw and write. Student's roles can be split up as follows: ● Recorder - writes on WB	Create 'whiteboards' using a digital tool. You can use some of the options listed below. Ask students to screenshot their lab work, upload it to their whiteboards, and write additional details. Instruct students on how to take a screenshot and upload.

	• Presenter - Starts the discussion about WB • Leader - make sure everyone agrees	Student's roles can be split up as follows: • Recorder - Shares Screen and writes on WB • Presenter - Starts the discussion about WB • Leader - make sure everyone agrees, uploads the board
Commenting and discussing the class' whiteboards	Have students walk around the classroom and leave sticky notes on other group's whiteboards with comments, questions, and thoughts. Have students review comments they received on their whiteboards.	All of the digital whiteboards options below allow students to view their classmate's boards. Students can also screenshot their work and paste it into a Slides/Powerpoint document in order to have all boards in one place. You can ask students to view other people's whiteboards and leave comments with methods specific to whichever whiteboard you choose.
Having a class-wide whiteboard meeting	Reconvene as a class for a discussion. When working on definitions or needing to document class work, you can use the slides we provide in this curriculum and project them to the front of the room. If you happen to be working without a projector, you can document on your whiteboard and take pictures at the end of class (however we would recommend digital documentation so that you can revisit with your class throughout). There are moments in the curriculum in which we will ask students to share their questions with the class and then see if they can answer any of the other	Reconvene on your video-conferencing platform and have a full discussion. Use the slides provided in the curriculum or create your own slides/documents to keep track of your class' thoughts during discussion. Documenting in this way will make lessons easier to revisit. There are moments in the curriculum in which we will ask students to share their questions with the class and then see if they can answer any of the other students' questions. For this activity, you can ask students to write their questions in the chat and then respond in the chat. You could also use some of the chat discussion options provided below.

	students' questions. For these activities, you can have students post sticky notes at the front of the class then, in smaller groups, cycle through and have students respond to the questions.	

Options for Digital Whiteboards

- Whiteboard option 1 - Jamboard
    - Each group can make one slide and click through to view other slides. Students can write, draw, upload an image, change the background, etc.
    - Jamboard is entirely free with no limits on functionality
- Whiteboard option 2 - https://www.whiteboard.chat/
    - Slightly fancier features than Jamboard but may require a slightly more preparation. Every group can make their own whiteboard and click out to see other boards. Includes teacher tools like a grid view of everybody's board, option to project boards to the class
    - In the free version, teachers have ads displayed to them (but students do not). Free version is limited to 10 boards at a time (which should suffice for group work)
- Whiteboard option 3 - Miro's Web White Board
    - Slightly improved features from those of Jamboard but not everything is unlocked in the free version. Every group can work in the same space and then zoom in to other people's boards when they're ready for a gallery walk.
    - Requires upgrades for certain features

Options for Digital Discussions

- Option 1: https://yoteachapp.com/

    - The website serves as a kind of chat room that allows students to send messages, respond to specific messages, and even draw/graph/upload responses. While the interface is a little messy, it's a nice tool for keeping track of discussions and their responses.
- Option 2: Google sheets
    - You can create a google sheet to have students type their questions. Responses can be in the form of comments or indented/bulleted responses.
- Option 3: Jamboard
    - Have students write their questions in one color sticky note and their answers in another color. You can organize questions in columns or clusters. Here's a sample of a discussion board in Jamboard.

Instructional Goals:

6th Grade:

1. Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).
2. Solve unit rate problems including those involving unit pricing and constant speed.
3. Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in real-world context.

7th Grade:

1. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship b. Identify the constant of proportionality (unit rate) c. Represent proportional relationships by equations.
2. Explain what a point (x, y) on the graph of a proportional relationship means in the situation, with special attention to the points (0, 0) and (1, r) where r is unit rate.

8th Grade:

1. Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
2. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
3. Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.

Lesson Sequence:

Section 0.0 - Supplemental Materials Checklist

Section 1.0: Perfect Purple Paint (Ratios and Proportions)

1.1 Introduction

1.2 Create a Model

1.3 Refine Your Model (Whiteboard Discussion)

- Suggested Assignment: Intro to Ratios
- Suggested Assignment: Help Nico with Ratios

1.4 Practice Help Giving & Review Talk Moves

1.5 Create Your Own Color

1.6 Discuss your Model

1.7 Integrate Feedback on Model

- Suggested Assignment: Proportional Relationships
- Suggested Assignment: Solving Proportions

Section 2.0: Buggy Lab (Slope and Speed)

2.1 Proportion Problems Brainstorm

2.2 Accuracy with a Stopwatch

2.3 Buggy Lab: Collect Data

2.4 Buggy Lab: Create a Whiteboard

2.5 Buggy Lab: Discussion

2.6 Buggy Lab: Board Meeting

- Suggested Assignment: Khan Academy Unit Rate

Section 3.0: Row Boats (Linear Equations)

3.1 Row Boat Lab: Collect Data and Make Whiteboards

3.2 Row Boat Lab: Discuss Your Models

3.3 Row Boat Lab: Whiteboard Meeting

3.4 Row Boat Lab: Takaways

- Suggested Assignment: Help with Functions Assignment
- Suggested Assignment: Slope-Intercept Form

Section 0.0 Materials Checklist

Links labeled with lesson numbers are specific to those lessons. Links without a number are used throughout or are suggested materials.

Physical Materials:

- ☐ Whiteboards
- ☐ Sticky Notes
- ☐ Computers (can be one per student, per group, etc. use your best judgment)
- ☐ Unifix cubes (can also be digital if you do not have a physical set)
- ☐ Tumble Buggies
- ☐ Meter Sticks
- ☐ Masking tape
- ☐ Markers
- ☐ Erasers
- ☐ Stopwatch (students can also use their cell phone stopwatch)

Teacher resources:

- [ ] Image of group norms
- [ ] Modeling intro for teachers
- [ ] Talk moves teacher sample
- [ ] 1.0_Teacher Check Sheet for Paint Splash WB meeting
- [ ] 3.0_Teacher Answers for help with Functions Assignment

Slides:

- [ ] Slides for Class/Study Norms
- [ ] Model-So-Far Slides
- [ ] 2.0_Buggy lab instruction slides
- [ ] 3.0_Boat lab instruction slides

Digital Tools:

- [ ] Whiteboard option 1 - Jamboard
- [ ] Whiteboard option 2 - https://www.whiteboard.chat/
- [ ] Whiteboard option 3 - Miro's Web White Board
- [ ] Digital Discussion Option 1: https://yoteachapp.com/
- [ ] Digital Discussion Option 2: Google sheets
- [ ] Digital Discussion Option 3: Jamboard
- [ ] Digital graphing tool for linear equations
- [ ] 1.0_Unifix cubes online
- [ ] 1.0_Proportion Playground - Paint Splash
- [ ] 2.0_Online stopwatch
- [ ] 2.0_Digital tool for bar charts

- [ ] 2.0_Buggy YouTube Video_Option1
- [ ] 2.0_Buggy YouTube Video_Option2
- [ ] 3.0_Student Link to Graphing Eileen's Motion Lab (Boat Lab)
- [ ] 3.0_Teacher Link to Boat Lab

Khan Academy Videos:

- [ ] 1.0_Khan Academy - Finding Ratios: An Introduction
- [ ] 1.0_Khan Academy - Introduction to Proportions
- [ ] 1.0_Khan Academy - Solving Proportions
- [ ] 2.0_Khan Academy - Graphing proportional relationships - unit rate
- [ ] 3.0_Khan Academy - Intro to slope-intercept form (y=mx+b) | Algebra
- [ ] Khan Academy - Intro to slope | Algebra (supplement, not used in curriculum)
- [ ] Khan Academy - Linear Equations Graphs: word problems (supplement, not used in curriculum)
- [ ] Khan Academy - Modeling with Linear Equations: snow (supplement, not used in curriculum)

Student Handouts:

- [ ] Talk move student sample
- [ ] 1.0_Intro to Modeling for Students
- [ ] 3.0_Linear equation graphic organizer

List of Assignments

- [ ] Test Option 1 (you are welcome to use this as a pre or post-test)
- [ ] Test Option 2 (you are welcome to use this as a pre or post-test)

1.0 Perfect Purple Paint:

- [ ] 1.0 Full Packet of Student Assignments including:
    - [ ] 1.0_Talk Move Assignment
    - [ ] 1.0_Ratios Assignment
    - [ ] 1.0_Help with Ratios Assignment
    - [ ] 1.0_Proportional Relationship Assignment
    - [ ] 1.0_Solving Proportions Assignment

2.0 Buggy Lab

- [ ] 2.0 Full Packet of Student Assignments including:
    - [ ] 2.0_Buggy Lab Worksheet
    - [ ] 2.0_Unit Rate Assignment

3.0 Row Boat Lab

- [ ] 3.0 Full Packet of Student Assignments including:
    - [ ] 3.0_Boat Lab Assignment
    - [ ] 3.0_Help with Functions Assignment
    - [ ] 3.0_Intro to Slope Intercept Form Assignment

Useful Links (not critical to curriculum)

- [ ] Image of math group norms
- [ ] Didax Virtual Manipulatives Teaching Tools

1.0 Perfect Purple Paint (Ratios and Proportions)

“Ratios and proportions are foundational to student understanding across multiple topics in mathematics and science. In mathematics, they are central to developing concepts and skills related to slope, constant rate of change, and similar figures, which are all fundamental to algebraic concepts and skills. Ratios and proportions are used in relationships found in triangles, including trigonometric ones, such as sine, cosine, and tangent, found in later algebraic instruction. In science, they are used when quantities involve density, acceleration, and other comparable derived measures. Even in real-life situations, ratios and proportions are useful when determining amounts to be used in recipes or finding the mileage per gallon of gas. In general, ratios and proportions describe relationships between and among quantities.”

- *Helping Students With Mathematics Difficulties Understand Ratios and Proportions. Barbara Dougherty, Diane Pedrotty Bryant, Brian R. Bryant, and Mikyung Shin. TEACHING Exceptional Children, Vol. 49, No. 2, pp. 96–105. Copyright 2017*

In this section, students develop an understanding of the concept of ratios and connect the numbers to the graph of the data. They can begin to understand the special proportional relationship that occurs when they graph the equivalent ratios and how to identify it as a straight line that passes through (0,0) and what makes it different from fractions or more random data. They are developing the understanding to “develop” a formula for linear equations.

Instructional Goals:

- **6th Grade**: Use ratio and rate reasoning to solve mathematical problems and problems in real-world context (e.g., by reasoning about data collected from measurements, tables of equivalent ratios, tape diagrams, double number line diagrams, or equations). a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed.
- **7th Grade**: Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of

proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations.

- **8th Grade**: Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. . *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- **8th Grade**: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Section 1.0 - Perfect Purple Paint (Ratios and Proportions)

1.1 Introduction
1.2 Create a Model
1.3 Refine Your Model (Whiteboard Discussion)
    Suggested Assignment: Intro to Ratios
    Suggested Assignment: Help Nico with Ratios
1.4 Practice Help Giving & Review Talk Moves
1.5 Create Your Own Color
1.6 Discuss your Model
1.7 Integrate Feedback on Model
    Suggested Assignment: Proportional Relationships
    Suggested Assignment: Solving Proportions

Suggested Lesson Breakdown

Pre-work: Introducing Help-giving, modeling, and class norms	**PPP: Unifix Whiteboard Activity**	**PPP: Paint Splash Whiteboard Activity**
Pretest (45 min)	**1.2 Create a PPP Model**	**1.4 Practice Help Giving & Review Talk Moves** Help Giving Assignment 1 (independent work of homework) and review
1.1 Introduction: Welcome, class norms, homework/grading explanation Introduction to Modeling	**1.2 Review other PPP Models** Gallery Walk	**1.5 Create Your Own Color**: Phet Simulation - Paint Splash
Introducing Help Giving (Talk moves expectations and assignment) (Independent)	**1.3 Refine Your PPP Model:** Whiteboard Discussion	**1.6 Discuss Your Model**: Small group chat
	Suggested Assignment: Intro to Ratios	**1.7 Integrate Feedback on Paint Splash Model:** Whiteboard Meeting
	Suggested Assignment: Help Nico with Ratios Assignment	**Suggested Assignment: Proportional Relationships Assignment**
	1.0 Full Packet of Student Assignments	**Suggested Assignment: Solving Proportions Assignment**

Pre-work: Introducing Help-giving, modeling, and class norms

Lesson	Text	Teacher Directions	Materials
PPP 1.1	**1.1 Introduction - Start here!** In the next three weeks, you will be interacting with your fellow classmates and others to learn and help teach some math concepts. We want to help you become better collaborators so: - You can support each other in your learning. - You can become more active learners. - You can construct your own STEM knowledge.		
Pretest	**Pretest**	You can use either of the tests linked here as a pretest and use the other test as a post-test.	Test Option 1 Test Option 2
PPP 1.1	**1.1 Class Norms** Let's have a discussion about your classes. How do you like class to work? Let's discuss. Now that we know your class norms and what you want from me, let's take a look at what we need from you in a modeling classroom.	Review your own class norms with the class (if you have them) and use the left side of the provided Norm slide to document expectations. If you don't have class norms, take this time to document expectations or feel free to use the provided ones as an example. Set behavior expectations. Ask the students what they want from you,	Image of group norms Slides for Class/Study Norms

		and write it on the left side of the slide before they click on the slide Let them know what you want from them. On the right side of the norms slide, we have modeling norms that you should feel free to use alongside the sample image of group norms.	
PPP 1.1	**1.1 Introduction to Modeling**	Go through the linked guide to modeling with your students Review the teacher introduction if needed	1.0_Intro to Modeling for Students Modeling intro for teachers
PPP 1.1	**1.1 Introducing Help Giving** It's really important to learn to help others. Not just for others learning, but yours too. Now is your chance to get familiar with some help-giving techniques. Complete the provided Talk Moves assignment. **After Assignment** Do you have any questions about the Help-Giving and Talk Moves Assignment? Do you have any questions about anything so far?	In the help giving scenarios assignment, students pick best move and fill in an answer **Conversation starter:** "What did you think the best answer was for the first scenario?" **Expectations:** There is no one "best" talk move for a particular situation. **Conclude there is no one answer.** You can be helpful without completely	1.0_Talk Move Assignment Talk moves teacher sample Talk move student sample

	Now's your chance to discuss! Write your questions for the class to read (see digital/physical instructions in introduction for clarification). Look at the questions. Are there any questions posted about the Help-Giving and Talk moves Assignment? Can you give any help to your classmates?	knowing the content. You can review sample talk moves and their goals in the teacher copy and, if you'd like, distribute a student copy to your students You can save part of this assignment to review when you circle back to talk moves in PPP: Paint Splash Whiteboard Activity	

PPP: Unifix Whiteboard Activity

Lesson	Text	Teacher Directions	Materials
PPP 1.2	**1.2 Create a PPP Model** I've found the perfect color to paint my house. It's made by mixing two cups of blue paint with three cups of red paint. That makes one batch of Perfect Purple Paint. I know it will take at least 20 cups to paint my bathroom. I don't want to make a bunch of small batches. How much of each color will I need to paint the entire bathroom? Let's make a model using Unifix Cube blocks to represent the information in this problem.	Group work, break up students responsibilities as follows (specifics related to digital/physical learning provided in the introduction): • Recorder • Presenter • Leader **In-person:** If you have physical unifix cubes, you can conduct the activity with those. Otherwise, you can have students conduct the experiment on classroom devices (either one per student or per group).	1.0_Unifix cubes online Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard option 3 -

	Have one color block represent red paint and another color block represent blue paint. Show how you could make 20 cups of purple paint using the Unifix tool. **Explain your PPP Model** Now your group will use a whiteboard to explain your thinking. Everyone needs to agree with and understand everything that goes on the group board. As you make your whiteboard, think about these questions: ● How do you know the color will be the same in the small batch as it is in the larger batch? ● Can you find multiple ways to model this relationship? What if we only needed enough paint for one purple accent wall? ● Can you write a rule (using words or numbers) to create this color of paint that anyone could follow for any number of cups? ● Explain your thinking in words and draw arrows or labels to help others understand your thinking	**Digital:** Have students conduct the experiment on their personal devices using the unifix link. **Review Ratio Misconceptions:** ● Ratios amounts are often confused with fractions involving the same digits. For instance 2 : 3 is confused with ⅔ or 1 : 2 = ½. ● When solving problems involving proportion students tend to struggle with forming a ratio. For instance, 3 apples cost 45 cents would form the ratio apples : cost. ● When writing ratios into the form 1 : n students incorrectly assume that n has to be an integer or greater than 1. ● Ways to represent ratios: Drawing, using colon (:), the word “to”, ● Does not mean ⅔ two out of three. Not a fraction!!! This would mean 2 out of the three cups, not 2 cups and 3 cups (total 5.) Teacher visits each group. For	Miro’s Web White Board

		struggling students ask: • Can you reduce ratios in the same way you reduce fractions? What's the difference? • Can you multiply the ratio and still keep the same color? For students that already understand ask: • Can you write a number sentence or equation that would work for any amount of paint? Your job is to prove that the color is consistent. How can you do that?	
PPP 1.2	**1.2 Review Other PPP Student Models** Gallery Walk: Look at other people's boards and leave constructive comments or ask helpful questions about other models. When the Gallery walk is over, each group needs to read and discuss their comments and be ready to answer your classmates' questions. Are there any changes you would like to make to your board before you defend your work?	You can **review talk moves** before you start this exercise and have a discussion similar to the day before. Model commenting for students: have students walk around and leave sticky notes on the whiteboards with comments and questions.	Talk move student sample If you're working digitally, use one of the provided tools to review other student's models and leave comments.

PPP 1.3	**1.3 Refine Your PPP Model** Let's discuss your whiteboards as a class in a "Board Meeting." Keep in mind the suggestions from the badges to ask good questions and make productive comments. 1. Would it help us to be able to visualize these numbers? What are ways we can see what's happening with data? 2. What is our data telling us? 3. How can you prove to me that it will make the same color? 4. Can we write a rule for this in words? We want to make sure we all agree on the terms we are using. Are there any words we need to define as a class? We can discuss terms using the **Vocabulary** section of the Model-So-Far page.	**Reminder:** DON'T Pre-define vocabulary - use another whiteboard in the meeting or the model-so-far slide to define vocabulary together. **Teacher Led Discussion:** • (If doing this digitally, teacher shares screen) • Discuss Ratio vs fraction • Use talk moves from earlier in class • Teacher graphs different student responses. Observe the linear relationship when the equivalent ratios are graphed. **Goal**: Identify/solve/define ratios **Whiteboard meeting questions:** What similarities do we see between all the groups? (ex. one goes up the other goes up, same ratio/reduces to the same number.) • What is our data telling us? • How can you prove to me that it will make the same color? When you graph the students' ratios, it will become clear when one of the data points is not on the line. A class graph	Digital graphing tool for linear equations Model-So-Far page for vocabulary

		of the data will be part of the proof that the color will be the same. (See below) Use one exemplar whiteboard to show table, graph, sketch, description ● Discuss unit rate ● Write a rule in words We seemed to have noticed there is a special relationship between the red and blue paint. They all reduce to ⅔. Anyone seen a relationship like this before? This relationship is called a proportion. Based on our observations, what do we think are the characteristics of a proportion? How can we make more sense of our numbers (elicit response: make a table, make a graph.) Draw out the importance of using a diagram, labels, and explanation to get their point across. <u>Use a whiteboard to create a graph that uses the ratios from every group. This will make the linear relationship created by the ratio with the data the students calculated.</u>	

		Ask students: Looking at this graph of our data, if I had 9 cups of blue paint how many red cups would I need to get my perfect purple color? (Use some #'s that aren't already plotted) (Have students sketch a conceptual graph on their whiteboard to refer back to later) You can have students take a photo of the whiteboard and add it to a google sheet to have easy access to all boards for reference later. You can have students erase all whiteboards at this time.	
Suggested Assignment	**Suggested Assignment: Intro to Ratios** If you need help with ratios, watch the Khan Academy Finding Ratios: An Introduction	If your students need help with ratios, you can tack on the Khan Academy video as homework or classwork. **Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	1.0_Khan Academy - Finding Ratios: An Introduction 1.0_Ratios Assignment

	Suggested Assignment: Help Nico with Ratios Assignment When you are teaching someone else, it is often helpful to know the process and the answers before you begin. Use this assignment to be ready to defend your answers. You can also review **Talk Moves** to help you make constructive comments.	This assignment heavily scaffolds students through the process of solving the ratios while allowing them to practice help giving with talk moves.	1.0_Help with Ratios Assignment

PPP: Paint Splash Whiteboard Activity

Lesson	Text	Teacher Directions	Materials
PPP 1.4	**1.4 Practice Giving Help and Review Talk Moves: Assignment** Do you have any Questions about the Help Giving and Talk Moves Assignment? Do you have any questions about anything so far? Now's your chance to discuss! You can also review **Talk Moves** to help you make constructive comments.	You can save a few of the scenarios from the help giving assignment and circle back here to reinforce and remind students about the help giving and dialogue techniques. Review Help Giving and Talk Moves have students defend their choices, but be sure to reinforce the understanding that there is no "wrong" choice if it helps their	1.0_Talk Move Assignment Talk moves teacher sample Talk move student sample

		understanding or the understanding of the class. Conversation starter for the discussion: "What did you think the best answer was for the first scenario?" You can use the Bellwork slides to assist in your discussion. **Conclude there is no one answer.** You can be helpful without completely knowing the content.	
PPP 1.5	**1.5 Create Your Own Color**: **Phet Simulation - Paint Splash** I love my Perfect Purple Paint, but you probably want to come up with your own awesome color for your house. Check out Paint Splash here: https://phet.colorado.edu/en/simulation/proportion-playground Working as a group in Paint Splash in the single paint splash mode, create a paint splash of your choice. Your group needs to agree on the color. Each group member should then use the double paint splash setting to recreate your group's paint splash on the left. And then	Students will agree on a group color and create a table and graph. For group work, break up students responsibilities as follows (specifics related to digital physical provided in the introduction): • Recorder • Presenter • Leader **In-person:** Have students conduct the experiment on classroom devices (either one per student or per group). **Digital:** Have students conduct the experiment on their personal devices using the unifix link.	1.0_Proportion Playground - Paint Splash Digital graphing tool for linear equations Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard

	on the right create either a larger or smaller matching batch (each group member should create a different sized matching paint splash on the right). Enter all the group members' Ratios into the graphing tool. Remember, your whiteboard needs: ● A Title ● A picture of one group member's double paint splash (2 different sized batches) ● A picture of the table that lists your group members' batches ● A picture of your graph ● A written explanation about how you know how to find any size batch for your color. ● A rule in words and numbers.		option 3 - Miro's Web White Board
PPP 1.6	**1.6 Discuss Your Paint Splash Model: Gallery Walk** Look at other student's whiteboards and leave a comment on each picture. Ask a question for clarification, give a compliment, or explain what you are thinking. Be specific.	**1. Comments and Questions** Ask a Question Ask for clarification: "What do you mean by that?" "Can you give an example?"	Talk moves teacher sample Talk move student sample

	Review these **Talk Moves** to help you make constructive comments Do you understand / agree with their solutions? Use the talk moves to be clear and polite. Once you have commented individually, move to the group discussion: With your classmates, discuss the similarities and differences between the way groups represented the relationship between the red and blue paint? Does the way the group represented it change the color of the paint? How did people represent their perfect purple paint? Do different people's representations mean the same thing, and how? How are the representations (drawings, numbers, etc.) related?	Teacher makes comments to challenge the quality of student comments. **2. Analysis** If the way you write the ratio changes the color, what kind of rule can we define then it's not the same ratio. **Prompt -** Discuss the similarities and differences between the way groups represented the relationship between the color of the paint? Does the way the group represented it change the color of the paint? When students finish their discussion, have students look at the comments others made on their board. Ask: Do you want to make any changes before a Whiteboard meeting? Prompt - What are the similarities and differences between the graphs? What features do they share? Why do you think they share these features	

PPP 1.7	**1.7 Integrate Feedback on Model: Paint Splash Whiteboard Meeting** Review the comments your group received in the Whiteboard. Is there anything you would change on your Whiteboard Tool based on your classmates' feedback? What can you change to make your board more clear? If you do not yet have a rule on your board, do your best to add one. Think about how you would find the number of cups of each color needed for any size room. Let's discuss your whiteboards as a class. Keep in mind the suggestions from the talk moves to ask good questions and make productive comments. Make changes your group can all agree on. **Vocabulary** We want to make sure we all agree on the terms we are using. Are there any words we need to define as a class? We can discuss terms using the **Vocabulary** section of the Model-So-Far page.	While students are reviewing comments, try to encourage them to pull out the difference between a ratio and a fraction in the discussion. **Whiteboard meeting questions:** ● What similarities do we see between all the groups? (ex. one goes up the other goes up, same ratio/reduces to the same number.) ● What is our data telling us? ● How can you prove to me that it will make the same color? Project one exemplar whiteboard to show table, graph, sketch, description ● Discuss unit rate ● Write a rule in words There is a special relationship between Any color chosen. Has anyone seen a relationship like this before? All the quantities are proportional. Based on our observations, what do we think are the characteristics of a proportion? (you go up and over by the same amount on the graph, multiply by the unit rate, line passes through 0, line of a graph showing proportional	Talk moves teacher sample Model-so-far vocabulary Page

		relationship is a straight line) How can we make more sense of our numbers (elicit response: make a table, make a graph.) **Draw out the importance of using a diagram, labels, and explanation to get their point across.** DON'T Pre-define vocabulary (straight line/linear, goes through 0, same slope, same ratio, same unit rate) So what is the unit rate that we're seeing? Vocabulary to pull out: ratio, unit rate, proportion, (keep adding to definition in vocabulary brainstorm page as students make more connections. They don't need to see every piece at once.) A ratio is a comparison of two numbers A proportion is an equation that says two ratios are equivalent. Q: What does this table mean? What does the graph mean? A: The table is a list of ratios. The graph is a way of showing the	

		equation the says they are related. Explain why all the graphs deal with the first quadrant but omit the other three quadrants. Q: Why have we chosen to organize our data in this way? How does this help us understand our data? A: “Making Predictions” Q: How do you know if you’re looking at a graph of a proportional relationship? A: Passes through origin (0,0), straight line, when one goes up the other goes up. You don’t need to get students to a perfect understanding of proportion by the end of the WB meeting!!!	
Suggested Assignment	**Suggested Assignment: Proportional Relationships Assignment** We have just graphed two Proportional relationships. Watch the video on Proportional Relationship Assignment. Complete the assignment after watching this video on Khan academy to help you think about proportions.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else’s question.	1.0_Proportional Relationship Assignment 1.0_Khan Academy - Introduction to Proportions

Suggested Assignment	**Suggested Assignment: Solving Proportions Assignment** Watch the video about Solving Proportional Relationships. Complete the assignment after watching this video on Khan academy to help you solve proportions with a missing variable.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	1.0_Solving Proportions Assignment 1.0_Khan Academy - Solving Proportions

2.0 Buggy Lab (Slope and Speed)

In this section, students connect the proportional relationship of constant speed with the slope of the line. They can use the slope of a line to create the equation for the graph of their real world data and then use slopes to graph other lines and begin to understand the story of the graph even without having to collect the data themselves. At this point, students will have an understanding of "m," the first part of the linear equation y=mx+b.

Instructional Goals:

- **6th Grade**: Solve unit rate problems including those involving unit pricing and constant speed.
- **7th Grade**: a.Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in the situation, with special attention to the points (0, 0) and (1, r) where r is unit rate.
- **8th Grade**: Graph proportional relationships interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Section 2.0: Buggy Lab (Slope and Speed)

2.1 Proportion Problems Brainstorm
2.2 Accuracy with a Stopwatch
2.3 Buggy Lab: Collect Data
2.4 Buggy Lab: Create a Whiteboard
2.5 Buggy Lab: Discussion
2.6 Buggy Lab: Board Meeting
Suggested Assignment: Khan Academy Unit Rate

Suggested Lesson Breakdown

Buggy Lab Data Collection	Buggy Lab Discussion
2.1 Proportion Problems Brainstorm	2.5 Buggy Lab: Gallery walk
2.2 Accuracy with a Stopwatch	2.6 Buggy Lab: Board Meeting
2.3 Buggy Lab: Collect Data	Suggested Assignment: Khan Academy Unit Rate assignment
2.4 Buggy Lab: Create a Whiteboard	2.0 Full Packet of Student Assignments

Buggy Lab Data Collection

Lesson	Text	Teacher Directions	Digital Elements
Buggy Lab 2.1	2.1 Proportion Problems Brainstorm So far we have solved a painting problem with ratios and proportions. We wanted to keep the same color no matter what size batch we needed. Let's brainstorm types of problems we might solve using what we know about ratios and proportions.	Documentation: Try to document this discussion wherever you keep your class notes or on your class whiteboard. When the brainstorm is over tell students, “When we start our next activity, part of your job will be to prove to your classmates if the relationship we investigate is proportional or not.	

Buggy Lab 2.2	**2.2 Accuracy with a Stopwatch** We're going to play around with a stopwatch. You can also use a cellphone/clock for this activity. If you have access to a stopwatch/cellphone/clock with a second hand, have it available for timing in our next activity.	**Class activity:** Everyone starts and stops the stopwatch at the same time (about 10 seconds) , then we read off the times and make a simple frequency graph. You can make a horizontal bar chart type graph using a digital tool and projecting to the class) We should see almost no one get the same time. As a class, decide what is a reasonable time. We are not as accurate as we think we are. Remember, most of the math we are exploring was discovered before clocks were capable of such precise measurement. **In-person:** Have students conduct the experiment with stopwatches in the classroom or with their personal phones **Digital:** Have students conduct the experiment on their personal devices. You could also use an online stopwatch linked here.	Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard option 3 - Miro's Web White Board Digital graphing tool for linear equations 2.0_Digital tool for bar charts 2.0_Online stopwatch

Buggy Lab 2.3	**2.3 Buggy Lab: Collect Data** Question that we will explore: Is speed a proportional relationship? We will be collecting a series of ratios (different units) and comparing them and plotting them on a graph to see if there is a proportional relationship. We'll observe the moving object and see what we notice. What measurements should we take? ● What units will we use? ● Do we plan to collect more than one set of data? ● What procedures do we need to follow so that our data is as accurate and comparable as possible? **In-person Student Instructions:** 1. Release the buggy before the start line. Start the timer when it reaches the line. 2. One student calls out time every 5 seconds. Have another student put a piece of tape on the floor when the buggy is at each time.	**Common student misconception:** Fractions use the same units - parts of a whole. Ratios use different units - comparing two things. (Be careful about hammering this home too early. It makes it very clear speed is a ratio.) We want distance on the y-axis as our dependent variable, because we want the standard meters/second so we understand the speed. Where on the buggy do we record each second? When do we push start/stop? As students are working, plant the questions for discussion: What is speed? What's the relationship between position and time? You can use the slides to help guide the assignment. **In-person:** Follow the student instructions with a physical buggy. If	2.0_Buggy Lab Worksheet 2.0_Buggy YouTube Video_Option1 2.0_Buggy YouTube Video_Option2 2.0_Buggy lab instruction slides

	Only push stop on the clock at the end. 3. Measure the distance from the starting line to each tape marker on the floor. **Digital Student Instructions:** Pause the video every 5 seconds *after* the car passes the starting line and record the position of the car. - Buggy video coming soon	you are not able to obtain buggies, follow the digital instructions. **Digital:** Have students play then pause the video every 5 seconds *after the car passes the starting line and record the position of the car. (0 time when the car passes the line, 0 position is the starting line.) Record at least 5 coordinates for the graph. - Buggy video coming soon	

Buggy Lab 2.4	**2.4 Buggy Lab: Make a Whiteboard** As a group, create a whiteboard with as many representations of the data as you can show. Then, create a rule using words and an equation using the y = mx format and write it on your whiteboard The whiteboard should include: ● Data table - Time on x-axis and Position on y-axis ● Graph with units and labels ● The story of the car - What did you observe? How did it move? How do you know? ● Is this a proportional relationship? Be prepared to defend your thinking! ● A rule in words and numbers that describes this relationship Remember: this is real data. Your measurements are not perfect. What rule works "best" or makes the most sense.	Students collect data by recording the position of the car every five seconds. So the independent variable on the x-axis will be time, and the dependent variable on the y axis will be position*. **Common misconceptions:** Students can get confused about which axis shows distance. Some can imagine the car "driving" from left to right on their graph, and that is not what the graph is showing. It is an abstraction of speed. The speed is the ratio that occurs over and over again because there is constant velocity. We are drawing a "picture" of speed. ***Position vs. distance misconception.** Position is the location of an object relative to the origin. This will be more important once we go to section 3 of this curriculum and address data that doesn't begin at the origin and other quadrants of the graph. Distance is the total amount the object has moved and is	Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard option 3 - Miro's Web White Board Digital graphing tool for linear equations

		always a non-negative number. **Whiteboarding:** Encourage students to include multiple representations on their whiteboards: drawing/motion map, table, graph, words explaining their graph/motion. Etc., equation Students can add to their boards during a Whiteboard Meeting. All boards will look different. That's helpful for you as a teacher, because then there is something to discuss!	

Buggy Lab Discussion

Lesson	Text	Teacher Directions	Digital Elements
Buggy Lab 2.5	**2.6 Buggy Lab: Gallery walk** Look at the whiteboards of the other groups. Use Talk Moves to make helpful comments/questions.	Review **Talk Moves** with students to help them make constructive comments. **Discussion Questions to plant with students:** ● Every point on this line represents a ratio. What is a rule (equation) you can use to describe any point on this	Talk move student sample Talk moves teacher sample

	Make sure to think about how they have presented their information. You can also make comments about the following: 1. Every point on this line represents a ratio. What is the rate of change for the graph? Did the group create a rule (equation) you can use to describe any point on this line? 2. What does the rate of change on a graph mean? What does it mean specifically for this graph? What did we discover? 3. Is constant speed proportional? Explain your thinking. Do you agree with each board's rationale? Go back and look at the comments on your own whiteboard. Are there any changes you would like to make to your board before the whiteboard meeting?	line? ● Is constant speed proportional? Explain your thinking. Do you agree with each board's rationale? **Definition of rate**: When a ratio has different units it's called a rate - you can add this to another page of your Model-so-far vocabulary	
Buggy Lab 2.6	**2.6 Buggy Lab: Whiteboard Meeting** Think back to the notes you saw on your board or other boards you just observed. Did someone make a particularly good point you want to share? Did someone have a great way of explaining that helped you	**Guide the discussion to reach conclusions about vocab and add to the Model-so-far.**	Model-So-Far page

	understand? What did you discover from this activity? In the Board Meeting be prepared to discuss the following: ● Is constant speed proportional? ● What are we looking for to see if this relationship is proportional? ● What does the x represent in your Buggy Lab graph? What does the y represent in your graph? ● Why do we make graphs of data? ● Why do we write equations of lines? Then, we'll revisit the class definitions of Slope and Unit Rate in the **Vocabulary** page of **Model-So-Far**.	The commonly accepted definition of speed is position divided by time and the metric unit is the meters traveled by the object in 1 second (expressed with shorthand as m/s). Is every equation dealing with constant speed proportional? No, only graphs starting at the origin can be proportional. Describe the line using equations y=mx Slope = m = change in y/change in x Vocab: slope intercept form - y=mx+b in this case b will always be 0. How do you know you're looking at a proportional relationship? View the graph with the complete set of data points together. **Discussion goal:** We know this is a proportional relationship and a	

		series of ratios, but it's also a function. Some students think that all relationships that increase or decrease by a constant value are proportional. A proportional relationship is one kind of function. We will be learning about other kinds of functions next week.	
Suggested Assignment	**Khan Academy: Unit Rate Assignment** We now understand that we need a unit rate to make an equation for our data to make a graph. Complete Unit Rate Assignment using Khan academy to help you think about making equations. This will also give you the opportunity to give and receive help from students who are not in your class.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	2.0_Khan Academy - Graphing proportional relationships - unit rate 2.0_Unit Rate Assignment

3.0 Row Boat Lab (Linear Equations)

Once students have a strong understanding of ratios, proportional relationships, and how to read and create their graphical representations, it is much easier to help students build an understanding of linear relationships in general. By representing real data, they can apply their experience with proportional relationships to other data that is not necessarily proportional. Students will have a more intrinsic sense of what the data is representing as well as a greater real world connection and understanding of graphical representations of data.

Instructional Goals:

- **6th Grade:** Use variables to represent two quantities that change in relationship to one another to solve mathematical problems and problems in real-world context. Write an equation to express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation
- **7th Grade**: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
- **8th Grade**: Given a description of a situation, generate a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or a graph. Track how the values of the two quantities change together. Interpret the rate of change and initial value of a linear function in terms of the situation it models, its graph, or its table of values.

Section 3.0: Row Boats (Linear Equations)

Suggested Lesson Breakdown

Row Boat Lab	**Wrap Up**
3.1 Row Boat Lab: Collect Data and Make Whiteboards	**Suggested Assignment:** Help with Functions
3.2 Row Boat Lab: Discuss Your Models	**Suggested Assignment:** Slope-Intercept Form Assignment
3.3 Row Boat Lab: Whiteboard Meeting	**Post Test**
3.4 Row Boat Lab: Takeaways	3.0 Full Packet of Student Assignments

Row Boat Lab

Lesson	**Text**	**Teacher Directions**	**Digital Elements**
Row Boat Lab 3.1	**3.1 Row Boat Lab: Collect Data and Make Whiteboards** In the Buggy Lab we collected data which showed that the buggy's constant speed was a proportional relationship. Now, we will begin experimenting with changing one parameter at a time to see how it changes the graphs and the equations. We will also need to agree if our new relationships are proportional.	**Discussion Questions as students run the experiment:** • How does the slope change when the boat goes faster/slower? • Does the boat start at 0? What does that do to the line? • Can we Break the model? If it doesn't fit with what we know, there is something new going on. We are no longer working with a	3.0_Boat lab instruction slides 3.0_Teacher Link to Boat Lab 3.0_Student Link to Graphing Eileen's Motion Lab (Boat Lab) 3.0_Boat Lab Assignment

	You will run the experiment two times. The first time, just push start and collect your data without making any changes. The second time, you may click on the "Paddles" button and, as a group, choose one variable to change. Then collect your data a second time with the new parameter. How do you think this will change your graph? On your whiteboard, include: ● Table ● Graph ● An equation for each line ● How did changing the variable change the speed? ● How can you tell just by looking at the graph? At your equation? At your table? **Is every equation dealing with constant speed proportional? Whose boat is going the fastest?**	proportional relationship. Now we need to think more broadly about all linear relationships. ● Are **all** lines describing constant speed proportional? ● Who's boat is going fastest? (discuss absolute value) **Main takeaways:** ● When, why, how do we get a y-intercept other than 0 ● The more steep the line, the greater the slope. The greater the slope, the faster the boat. **In-person:** Have students conduct the experiment on classroom devices (either one per student or per group). **Digital:** Have students conduct the experiment on their personal devices	Digital graphing tool for linear equations Whiteboard option 1 - Jamboard Whiteboard option 2 - https://www.whiteboard.chat/ Whiteboard option 3 - Miro's Web White Board
Row Boat Lab 3.2	**3.2 Row Boat Lab: Gallery Walk**	Move around with the students to observe the boards and see where they are so far. Are there any groups	

	Look at other student's solutions and use a sticky note to leave a comment on each picture. Use talk moves to ask a question, give a compliment, or explain what you are thinking. Be specific. (Do you understand / agree with their solutions?) Once you have commented individually, move back to your group for a quick discussion. With your group members, discuss what you saw on other boards and see if there are any changes you want to make to your group's board before our board meeting.	that need a little direction or pointed questions about their representations or understanding at this point?	
Row Boat Lab 3.3	**3.3 Row Boat Lab: Whiteboard Meeting** Is every equation dealing with constant speed proportional? Whose boat is going the fastest? How do you know? What do we need to graph this boat's motion? Do we need more than a ratio to write an equation?	**Address common misconceptions:** ● *input vs output: "Let's discuss input and output. Point to an input on your board. How do you know that's an input? So how about an output? What does it mean when someone says there is only one output for every input? ● What does f(x) mean Often students think it means "f times x" when in reality, it means "f of x" ● How would you describe	

		what a function is to someone who hasn't heard of the word before? ● Graphs that are not linear can still be functions. Lines on a graph look different because relationships are different. ● Graphing the function helps us understand the "story" of the equation. Give us a way to visualize the relationship. **Definition of a function** 1. one input = one output 2. It has three parts ● an input (x) ● a relationship ● and an output (y) 3. A function ***relates*** an input to an output. 4. Sometimes a function has a name - f(x) 5. Sometimes it has no name - y=x^2 Every straight line is a graph of a linear equation.	

3.4 Row Boat Lab: Takaways	**3.4 Row Boat Lab: Takaways** We have worked with two types of functions: Proportional functions and linear functions. We need to summarize what we know about functions. Let's add to our Model-So-Far page as a class: ● When is an equation a function? When is it not a function? ● When is a function proportional? ● Where is the ratio in the function? ● Why do we need b? Why do we need more information than the relationship between the two variables (ratio)?	**Wrapping Up** When the whiteboard meeting ends, have students summarize the model individually in words. Then discuss. Add to the Model-So-Far vocabulary as you discuss. The goal is for students to have a complete understanding of y=mx+b as an equation, how changing the equation affects the graph, and how to move from equation to data to graph in any order. You can use the provided linear equation graphic organizer to structure your discussion.	Model-So-Far page 3.0_Linear equation graphic organizer - to help organize student thinking on linear equations

Wrap Up

Lesson	Text	Teacher Directions	Digital Elements

Suggested Assignment	**Helping with Functions** You have one more opportunity to help the computerized Student Cobi with a new assignment. Read through the problems and explain to Cobi how you would solve them	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	3.0_Help with Functions Assignment 3.0_Teacher Answers for help with Functions Assignment
Suggested Assignment	**Slope-Intercept Form Assignment** We now understand that we can understand a problem better by easily switching between data in a table, an equation and a graph using Slope-Intercept Form. Complete Slope-Intercept Form Assignment using Khan academy to help you think about making equations.	**Suggested help-giving activity:** if you are active on Khan Academy/have accounts, have your students comment on the Khan Academy video to either ask a question or answer somebody else's question.	3.0_Khan Academy - Intro to slope-intercept form (y=mx+b) | Algebra 3.0_Intro to Slope Intercept Form Assignment
Post-test		You can use either of the tests linked here as a pretest and use the other test as a post-test.	Test Option 1 Test Option 2

Additional Khan Academy Videos

- Khan Academy - Intro to slope | Algebra (supplement, not used in curriculum)
- Khan Academy - Linear Equations Graphs: word problems (supplement, not used in curriculum)
- Khan Academy - Modeling with Linear Equations: snow (supplement, not used in curriculum)